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Nonuniform Electric Fields in Pulse Heated Wires[1]

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The influence of nonuniform electric fields on the measurement of the electrical conductivity in pulse-heated wires is studied analytically. Two causes for nonuniformity are considered: switching-on of an external voltage source (transient skin effect) and temperature-dependent change of the electrical conductivity. This problem usually has two strictly different time scales: a short one, on which the transient skin effect takes place, and a long one, on which heat conduction happens. Here, the short time scale is considered.

KEY WORDS: electrical conductivity; measurement error; nonuniform fields; pulse heating; skin effect.

1. INTRODUCTION

Electrical conductivity measurements during short pulse heating experiments usually assume [1], that voltage U and current I along the wire of length l and radius a are related to the electrical conductivity σ according to

$$\frac{U(t)}{I(t)}=\frac{1}{\sigma(t)}\frac{l}{\pi a^2} \tag{1}$$

However, the use of Eq. (1) implicitly supposes uniformly distributed electric fields and current densities across the wire. On the other hand, according to Faraday's law, quick changes of electric fields, which are

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inevitable in pulse-heating experiments, result in additional induction fields and eddy currents disturbing any spatial uniformity. Well known is the skin effect after switching-on of a current. Because of its opposite direction, the inductively generated fields reduce in this case the originally homogeneous field continuously from the surface to the center of the wire. Besides switching-on, there are further origins for time dependent variations and thus nonuniform electric fields during pulse-heating experiments:

- change of the electrical conductivity due to temperature and phase change of the sample,
- enlargement of the wire cross section because of volume expansion, and
- magnetic pinch and convection of the molton material because of Lorentz forces between current filaments in the wire.

Time-dependent variations of fields and the resulting skin effect in wires have been studied mainly in connection with sinusoidally alternating currents and its initial switching-on [2–4]. However, the boundary conditions used in these calculations render an immediate application of these results on pulse-heating experiments, as, e.g., described in Ref. 1, difficult. This holds even for the work of Phung and Miles [5], which studies the transient skin effect problem under consideration of a time-dependent conductivity. Here, however, the numerical treatment of this problem complicates a direct use of the reported data.

The present paper is an onset to treat the skin effect in pulse-heated wires under consideration of a time varying electrical conductivity analytically. Despite its inherent nonlinearity, an analytical treatment is possible, because the problem contains generally two different time scales: a short time scale at the beginning, on which field variations due to switching-on are strong but on which those due to conductivity changes are weak, and a long time scale, on which field variations due to conductivity changes play the important role but on which those due to the transient switching-on have already died out. Here we study the transient, or short, time scale effects only.

The aim of this paper is to give an estimate of the systematic error that occurs when Eq. (1) is used for the determination of the electrical conductivity in pulse-heating experiments and to discuss the experimental conditions where this error is negligible. The applied boundary and initial conditions try to model as well as possible the experimental situation described in Refs. 1, 3, and 6. In the following sections, we report only the essential mathematical results; details of the calculations will be given elsewhere.

2. FORMULATION OF THE PROBLEM

The general experimental situation is shown in Fig. 1. A wire-shaped sample, whose radius a is much smaller than its length l,

$$a \ll l \tag{2}$$

is fixed along the axis of a cylindrical experiment chamber of radius b. A time-dependent voltage source $U(t)$ feeds a current into an electric circuit consisting of the sample and the chamber walls. For the sake of simplicity, the electrical resistance of the walls is considered to be negligible compared to that of the wire. We also assume that the influence of other conductors

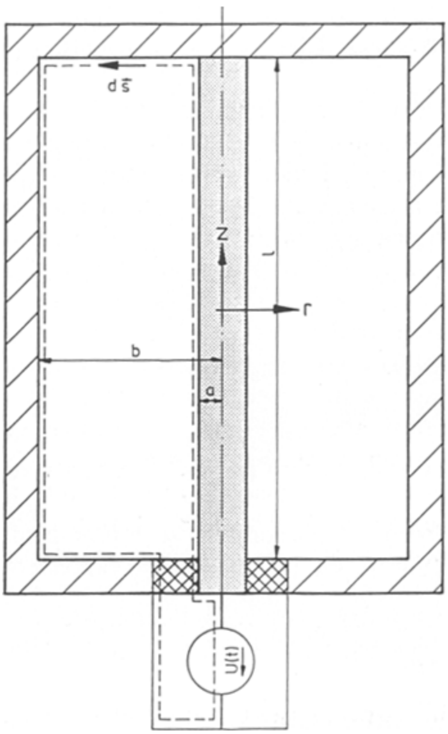

Fig. 1. Experimental arrangement showing the coordinates and the distances. The pulse-heated wire (shaded area) is fixed at the center of a cylindrical experiment chamber of electrically conducting (hatched) and nonconducting (cross-hatched) walls. A time-dependent voltage source, $U(t)$, is applied to the wire.

in the circuit can be neglected. Due to the consideration of a time-varying electrical conductivity $\sigma(T(\mathbf{x}, t))$, whose time dependence results from the time dependence of the temperature $T(\mathbf{x}, t)$, we have to account for the electrodynamic as well as thermodynamic behavior of the system.

2.1. Electrodynamic Description

The behavior of the electric field $\mathbf{E}(\mathbf{x}, t)$, magnetic field $\mathbf{B}(\mathbf{x}, t)$, and current density $\mathbf{j}(\mathbf{x}, t)$ in the wire is sufficiently well described by the quasistationary Maxwell equations, in which the space charge and the displacement current term are neglected, and the usual Ohm's law [7]

$$\begin{aligned} &\operatorname{rot} \mathbf{E}(\mathbf{x}, t) = -\partial/\partial t\, \mathbf{B}(\mathbf{x}, t), \qquad &&\operatorname{div} \mathbf{E}(\mathbf{x}, t) = 0 \\ &\operatorname{rot} \mathbf{B}(\mathbf{x}, t) = \mu_0 \mathbf{j}(\mathbf{x}, t), &&\operatorname{div} \mathbf{B}(\mathbf{x}, t) = 0 \\ &\mathbf{j}(\mathbf{x}, t) = \sigma(T(\mathbf{x}, t))\, \mathbf{E}(\mathbf{x}, t) \end{aligned} \tag{3}$$

This set of equations holds as long as the characteristic time scale, the order of magnitude of which is estimated at the end of this section, is long compared to the inverse plasma and mean collision frequency of the electron gas in the metal. Taking the rotational symmetry of the experimental arrangement and the condition of Eq. (2) into account, one obtains from Eq. (3)

$$\begin{aligned} &\mathbf{E}(\mathbf{x}, t) = E(r, t)\mathbf{e}_z, \qquad &&\mathbf{B}(\mathbf{x}, t) = B(r, t)\mathbf{e}_\varphi, \\ &\mathbf{j}(\mathbf{x}, t) = j(r, t)\mathbf{e}_z, &&T(\mathbf{x}, t) = T(r, t) \end{aligned} \tag{4}$$

when cylindrical coordinates with the z-axis along the center of the wire are used.

The basic differential equation for the electric field in the wire can be derived immediately from Eq. (3) and can be expressed using Eq. (4) as

$$\left\{\frac{1}{\rho}\frac{\partial}{\partial \rho}\rho\frac{\partial}{\partial \rho}\right\} E(\rho, \tau) = \frac{\partial}{\partial \tau}\left\{\frac{\sigma(T(\rho, \tau))}{\sigma_0} E(\rho, \tau)\right\} \tag{5}$$

where the following dimensionless variables are introduced:

$$\rho = \frac{r}{a}, \qquad \tau = \frac{t}{a^2 \mu_0 \sigma_0} \tag{6}$$

Here, $\sigma_0 = \sigma(T_0)$ denotes the electrical conductivity at the beginning of an experiment, or experiment part (e.g., at room temperature or melting temperature).

Under consideration of the Maxwell equations [Eq. (3)] and the previous assumptions, the boundary conditions for the electrical field result from an integration $\oint \mathbf{E} \cdot d\mathbf{s}$ along the indicated path (dashed line), shown in Fig. 1, and from an integration $\oint \operatorname{rot} \mathbf{E} \cdot d\mathbf{s}$ along a circle around the wire surface. Assuming, again, only radial dependences of the fields, i.e., neglecting and effects, we finally obtain the boundary conditions

$$\left[E(\rho, \tau) + p \frac{\partial}{\partial \rho} E(\rho, \tau) \right]_{\rho = 1} = \frac{U(\tau)}{l} \tag{7}$$

for the electric field at the wire surface. Here, we used the notation

$$p = \ln(b/a) \tag{8}$$

Another requirement on $E(\rho, \tau)$ is its continuity inside the wire (especially at $\rho = 0$). As an initial condition we choose

$$E(\rho, \tau) \equiv 0 \qquad \Rightarrow \qquad U(\tau) \equiv 0 \qquad \text{for} \quad \tau < 0 \tag{9}$$

In the following, we assume that the order of magnitude of the differential operators appearing in Eqs. (5) and (7) is of ord(1). This means that according to Eq. (6), field variations occur on a characteristic length scale of ord(a) and a characteristic time scale of ord($a^2 \mu_0 \sigma_0$), which is typically $\approx 10^{-7}$ s; see, e.g., Ref. 6.

2.2. Thermodynamic Description

Since the electrical resistivity ρ_{el} of a metal, which is the inverse of the electrical conductivity σ, can, in a good approximation, be considered as a linearly increasing function of the temperature

$$\frac{\sigma_0}{\sigma(T)} = 1 + \Delta\rho_{\mathrm{el},0}(T - T_0) \tag{10}$$

where $\Delta\rho_{\mathrm{el},0} = \sigma_0 \, d\rho_{\mathrm{el}}/dT$, we have to account for the time-dependent temperature change during the experiment. Provided that convection can be neglected, this is described by the heat conduction equation [8], which, taking Eq. (6) into account, reads

$$\frac{\rho_{\mathrm{m}}(T)\, c_{\mathrm{p}}(T)}{a^2 \mu_0 \sigma_0} \left[\frac{\partial T(\rho, \tau)}{\partial \tau} - \kappa(T) \operatorname{div}_\rho \left\{ \frac{K(T)}{\tilde{K}} \operatorname{grad}_\rho T(\rho, \tau) \right\} \right]$$
$$= \sigma(T)\, E^2(\rho, \tau) - \dot{Q}(T)\, \delta(\rho - 1) \tag{11}$$

The material specific quantity $\kappa(T)$ is defined as

$$\kappa(T) := \frac{\tilde{K}\mu_0\sigma_0}{\rho_m(T)\, c_p(T)} \tag{12}$$

where $\rho_m(T)$ and $c_p(T)$ are the mass density and the specific heat, respectively, and where $\tilde{K}$ is the characteristic thermal conductivity in the considered temperature range. The heat loss from the surface is denoted by $\dot{Q}$. For example, for tantalum, $\kappa(T) < 3 \times 10^{-4}$ over the entire reported temperature range [6]. Together with our previous assumptions about the order of magnitude of the differential operators [ord(1)], this estimation means that on the short time scale ($\approx 10^{-7}$ s) of the switching-on effect, we consider here, there is practically no heat conduction in the tantalum wire and, consequently, also no heat loss from its surface. Therefore, we neglect the corresponding terms in Eq. (11) and obtain

$$\chi^{-2}(T)\, \Delta\rho_{\mathrm{el},0} \frac{\partial}{\partial\tau} T(\rho,\ \tau) = E^2(\rho, \tau) \tag{13}$$

with

$$\chi^{-2}(T) = \frac{\rho_m(T)\, c_p(T)}{\sigma(T)\, \sigma_0\mu_0 a^2\, \Delta\rho_{\mathrm{el},0}} \tag{14}$$

The initial condition is supposed to be $T(\rho, 0) = T_0$. For metals, at least for tantalum, the relative temperature change of the resistivity, i.e., $\Delta\rho_{\mathrm{el},0}$, is by far greater than the corresponding quantity of mass density or specific heat. Furthermore, the slight increase in the specific heat c_p with temperature is almost canceled by the slight decrease in the density ρ_m. Hence, the temperature dependence of $\chi^{-2}(T)$ is almost completely determined by that of $\sigma(T)$, which also assures that the right-hand side of Eq. (14) is positive. Therefore, we assume that

$$\chi^{-2}(T) = \chi_0^{-2}(1 + \Delta\rho_{\mathrm{el},0}(T - T_0)) \tag{15}$$

where, according to Eq. (14),

$$\chi_0^2 = \chi^2(T_0) = \frac{\sigma_0^2\mu_0 a^2\, \Delta\rho_{\mathrm{el},0}}{\rho_{m,0} c_{p,0}} \tag{16}$$

3. SOLUTIONS

In this section, only the highlights of the mathematical solutions are presented; the details will be given elsewhere. At first, let us consider the solution of the heat balance equation, Eq. (13), for the present case. Under consideration of Eq. (15) and the initial condition, it can immediately be integrated. Inserting this result into Eq. (10), we obtain the time-dependent conductivity

$$\frac{\sigma(T(\rho, \tau))}{\sigma_0} = \left(1 + 2\chi_0^2 \int_0^\tau E^2(\rho, \tau')\, d\tau'\right)^{-1/2} \tag{17}$$

According to the boundary conditions given by Eq. (7), the order of magnitude of the electric field is $E(\rho, \tau) = \mathrm{ord}(U(\tau)/l)$. Consequently, the integral term on the right-hand side of Eq. (17) is of the order of magnitude of

$$\varepsilon = (\chi_0/l)^2 \int_0^1 U^2(\tau')\, d\tau' \tag{18}$$

Provided that ε is "small enough," about $\varepsilon \lesssim \frac{1}{2}$, the right-hand side of Eq. (17) can be expanded in a power series,

$$\frac{\sigma(T(\rho, \tau))}{\sigma_0} = 1 - \chi_0^2 \int_0^\tau E^2(\rho, \tau')\, d\tau' + \cdots \tag{19}$$

Note that, with l and a from Ref. 6, and for tantalum at room temperature, we find that $(\chi_0/l)^2 = 7.6 \times 10^{-7} V^{-2}$.

With Eq. (19) the final version of Eq. (5) then reads

$$\left\{\frac{1}{\rho}\frac{\partial}{\partial \rho}\rho\frac{\partial}{\partial \rho}\right\} E(\rho, \tau) = \frac{\partial}{\partial \tau}\left\{E(\rho, \tau)\left(1 - \chi_0^2 \int_0^\tau E^2(\rho, \tau')\, d\tau' + \cdots\right)\right\} \tag{20}$$

As already mentioned, the integral term in Eq. (20) is of $\mathrm{ord}(\varepsilon)$. This suggests to solve the nonlinear field equation by expansion in powers of ε [9],

$$E(\rho, \tau) = \varepsilon^0 E_0(\rho, \tau) + \varepsilon^1 E_1(\rho, \tau) + \cdots \tag{21}$$

Provided that the experimental conditions result in a sufficiently small ε, which is often satisfied, the truncation of the series after the first order usually gives a good approximations. Substituting Eq. (21) into Eqs. (20) and (7), and comparing the coefficients of ε^n, we find for the order ε^0

$$\begin{gathered} \left\{\frac{1}{\rho}\frac{\partial}{\partial \rho}\rho\frac{\partial}{\partial \rho}\right\} E_0(\rho, \tau) = \frac{\partial}{\partial \tau} E_0(\rho, \tau) \\ \left[E_0(\rho, \tau) + \rho\frac{\partial}{\partial \rho} E_0(\rho, \tau) - \frac{U(\tau)}{l}\right]_{\rho=1} = 0 \end{gathered} \tag{22}$$

and for the order ε^1

$$\left\{\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}\right\}E_1(\rho,\tau)=\frac{\partial}{\partial\tau}E_1(\rho,\tau)-\frac{\chi_0^2}{\varepsilon}\frac{\partial}{\partial\tau}\left\{E_0(\rho,\tau)\int_0^\tau E_0^2(\rho,\tau')\,d\tau'\right\}$$
$$\left[E_1(\rho,\tau)+p\frac{\partial}{\partial\rho}E_1(\rho,\tau)\right]_{\rho=1}=0 \tag{23}$$

Both partial differential equations together with the proper boundary conditions are linear and can be calculated by standard mathematical methods. The complete solution of the first one is

$$E_0(\rho,\tau)=\frac{U(\tau)}{l}-\frac{1}{l}\sum_{k=1}^{\infty}\frac{2J_0(\lambda_k\rho)\int_0^\tau \dot{U}(\tau')\exp\{-\lambda_k^2(\tau-\tau')\}\,d\tau'}{\lambda_k J_1(\lambda_k)(\lambda_k^2p^2+1)} \tag{24}$$

where J_0 and J_1 are the zeroth- and first-order Bessel functions, respectively, and the numbers λ_k denote the positive real roots of

$$J_0(\lambda)-p\lambda J_1(\lambda)=0 \tag{25}$$

The time dependence of the external voltage $U(\tau)$ is determined by the experimental conditions. High-capacity power supplies are well described by a voltage jump from zero to a constant value U_0 at $\tau=0$. For this case we find

$$E_0(\rho,\tau)=\frac{U_0}{l}\left[1-\sum_{k=1}^{\infty}\frac{2J_0(\lambda_k\rho)\exp\{-\lambda_k^2\tau\}}{\lambda_k J_1(\lambda_k)(\lambda_k^2p^2+1)}\right] \tag{26}$$

Equation (26) is represented in graphical form in Fig. 2. A more complicated situation occurs for a current-dependent voltage, e.g., $U(\tau)=RI(\tau)+1/C\int_0^\tau I(\tau')\,d\tau'$, which results from a power supply of low capacity C and high internal resistance R. Together with the definition $I_0(\tau)=2\pi a^2\sigma_0\int_0^1 E_0(\rho,\tau)\rho\,d\rho$ for the current of order ε^0, Eq. (24) finally results in a linear integral equation for $I_0(\tau)$,

$$R_0 I_0(\tau)=\frac{1}{C}\int_0^\tau I_0(\hat{\tau})\,g(\tau-\hat{\tau})\,d\hat{\tau}+R\int_0^\tau I_0(\hat{\tau})\,g'(\tau-\hat{\tau})\,d\hat{\tau} \tag{27}$$

where g' denotes the derivative of

$$g(\tau-\tau')=4\sum_{k=1}^{\infty}\frac{1-\exp\{-\lambda_k^2(\tau-\tau')\}}{\lambda_k^2(\lambda_k^2p^2+1)},\qquad R_0=\frac{l}{\sigma_0\pi a^2} \tag{28}$$

With the detailed knowledge of $E_0(\rho,\tau)$, we are able to calculate $E_1(\rho,\tau)$ from Eq. (23). However, since this solution is relatively cumbersome in full generality, we consider here its asymptotic approximation for $\tau \to \infty$ only. On the other hand, because of the exponential decrease in time, this result becomes significant already after a few τ. Assuming a jump of $U(\tau)$ at $\tau = 0$ (see above), we obtain for the electric field up to the first order,

$$E(\rho, \tau \to \infty) = \varepsilon^0 E_0(\rho, \tau \to \infty) + \varepsilon^1 E_1(\rho, \tau \to \infty) + \varepsilon^2 \cdots$$
$$= \frac{U_0}{l}\left[1 + \chi_0^2 \left(\frac{U_0}{l}\right)^2 \frac{2p + 1 - \rho^2}{4} + \varepsilon^2 \cdots\right] \tag{29}$$

Since the zeroth-order term of the electric field does not consider any temperature dependence of the conductivity, its asymptotic value is just the constant U_0/l (Fig. 2). This is different for the first-order term. Even after the switching-on effects (transient skin effect) have disappeared for $\tau \to \infty$, there remains still a steady time-dependent change of the electrical fields and currents in the wire due to the continuously decreasing electrical conductivity with increasing temperature. The resulting decreasing current generates induced electric fields that point in the same direction as the

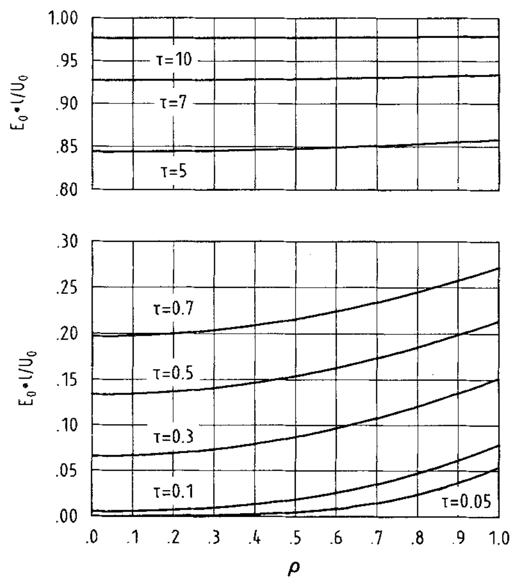

Fig. 2. Variation of the zeroth-order electric field, normalized to the constant external field U_0/l, as function of the normalized radial distance ρ and the normalized time τ for $p = 5$ according to Eq. (26).

original ones. This is just inverse of the situation occurring during the switching-on. Consequently, as reflected by Eq. (29), the electric field strength is higher at the center of the wire than at its surface (inverse skin effect).

4. SYSTEMATIC ERROR IN ELECTRICAL CONDUCTIVITY MEASUREMENTS

According to Ohm's law, Eq. (3), the electrical conductivity is, in the present case, defined by

$$\sigma(T(\rho, \tau)) = \frac{j(\rho, \tau)}{E(\rho, \tau)} \tag{30}$$

With the definition for current I and voltage U,

$$I(\tau) = 2\pi a^2 \int_0^1 j(\rho, \tau)\rho \, d\rho, \qquad U(\rho, \tau) = E(\rho, \tau)/l \tag{31}$$

an integration of Eq. (30) over the wire cross section results in Eq. (1), but only if the fields are spatially constant. As already mentioned, this condition is generally not satisfied during pulse-heating experiments. Nevertheless, since the only measurable quantities are the time-dependent current through the wire $I(\tau)$ and the time-dependent voltage drop at the surface of the wire $U(1, \tau)$, it is usually assumed that the quantity

$$\tilde{\sigma}(\tau) = \frac{l}{\pi a^2} \frac{I(\tau)}{U(1, \tau)} \tag{32}$$

at least approximates the real electrical conductivity for temperatures measured at the wire surface $\sigma(T(1, \tau))$; see, e.g., Refs. 1 and 6. In the following, we estimate the relative difference between the measured and the real (calculated) conductivity for this temperature:

$$\Delta(\tau) = \left| \frac{\tilde{\sigma}(\tau) - \sigma(T(1, \tau))}{\sigma(T(1, \tau))} \right| = \left| \frac{j(1, \tau)\, \pi a^2}{I(\tau)} - 1 \right| \tag{33}$$

Together with Eqs. (30) and (19), Eq. (21) implies an expansion in powers ε^n also for the current density,

$$\begin{aligned} j(\rho, \tau) &= \varepsilon^0 j_0(\rho, \tau) + \varepsilon^1 j_1(\rho, \tau) + \cdots \\ &= \sigma_0 E_0(\rho, \tau) + \varepsilon \sigma_0 \left[E_1(\rho, \tau) - \frac{\chi_0^2}{\varepsilon} E_0(\rho, \tau) \int_0^\tau E_0^2(\rho, \tau')\, d\tau' \right] + \cdots \end{aligned} \tag{34}$$

which, after integration according to Eq. (31), results again in a corresponding expansion for the current $I(\tau)=\varepsilon^0 I_0(\tau)+\varepsilon^1 I_1(\tau)+\cdots$. Inserted into Eq. (33), these expansions lead to

$$\Delta(\tau)\leqslant\varepsilon^0\Delta_0(\tau)+\varepsilon^1\Delta_1(\tau)+\cdots=\left|\frac{j_0(1,\tau)\,\pi a^2}{I_0(\tau)}-1\right|$$
$$+\,\varepsilon\left|\frac{I_1(\tau)}{I_0(\tau)}\left(\frac{j_0(1,\tau)\,\pi a^2}{I_0(\tau)}-\frac{j_1(1,\tau)\,\pi a^2}{I_1(\tau)}\right)\right|+\cdots \tag{35}$$

With the help of $j_0(\rho,\tau)$ from Eq. (34), and using for $E_0(\rho,\tau)$ the result of Eq. (26), which assumes a voltage jump at $\tau=0$, we find, after studying the bounds of the occurring sums, a relatively sharp estimation of the dominant error term $\Delta_0(\tau)$ by

$$\Delta_0(\tau)\leqslant\Delta_0^+(\tau)=\left[(2p+1)\left(\exp\left\{\frac{4\tau}{2p+1}\right\}-1\right)\right]^{-1} \tag{36}$$

The variation of $\Delta_0^+(\tau)$ is shown graphically in Fig. 3. For the typical value $p=5$, the relative error, which results from disregarding the skin effect, is below 1% after $\tau\approx 6.5$. The quantity p, Eq. (8), and thus the relation b/a (see Fig. 1) have an important influence on the decay of the skin effect. The greater the area enclosed by the electrical circuit, the greater is the penetrating magnetic flux, and the greater is the induced, oppositely directed

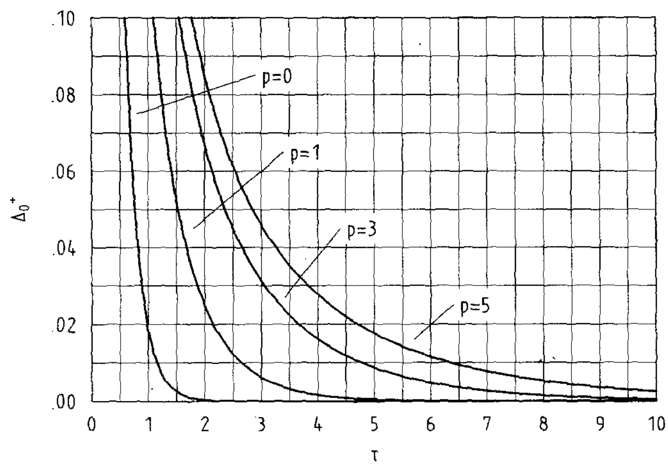

Fig. 3. Variation of $\Delta_0^+(\tau)$, which is an upper limit of the zeroth-order error term, as a function of the normalized time τ for different values of $p=\ln(b/a)$ according to Eq. (36).

electric field that delays in every filament of the wire the increase in current to its final value. Note, however, that $\Delta_0(\tau)$ do not account for the temperature- and thus time-dependent change of the electrical conductivity.

The zeroth-order term $\Delta_0(\tau)$ vanishes for $\tau \to \infty$. This is different for the corresponding first-order term $\Delta_1(\tau)$, which accounts for the temperature-dependent change of the electrical conductivity. The reason for this behavior has already been discussed (end of Section 3). Up to the first order in ε, the long-time behavior [on the time scale of Eq. (6)] of the systematic error can be estimated by

$$\Delta(\tau \to \infty) \leqslant \frac{5}{8} \chi_0^2 \left(\frac{U_0}{l}\right)^2 \equiv \frac{5}{8} \varepsilon \tag{37}$$

The constant χ_0 was defined in Eq. (16). Provided that the right-hand side of Eq. (37) is smaller than the tolerated measurement error, the remaining nonuniformity of the electric field for $\tau \gg 1$ has no significant influence on the measurement value of the electrical conductivity.

5. SUMMARY

We used analytical methods to study the influence of a nonuniform electric field on the measurement of the electrical conductivity in pulse heated wires. Two causes for nonuniformity have been taken into account: switching-on of an external voltage source and temperature-dependent change of the electrical conductivity. The simultaneous consideration of the Maxwellian and the heat conduction equations resulted in a nonlinear differential equation for the electric field. This has been solved by expansion in powers of ε [see Eq. (18)], which implicitly supposes that ε is a small quantity. From a practical point of view, this has been justified a posteriori by the result of Eq. (37), which shows that only a small ε keeps the nonuniformity of the electric field in the wire, and thus the systematic measurement error, small, after the transient skin effect has decayed.

We studied in the preceding sections the behavior of the electric field on a time scale $\text{ord}(a^2 \mu_0 \sigma_0) \approx 10^{-7}$ s, on which the transient skin effect due to switching-on takes place. This permitted the heat conduction in the wire to be neglected since it is a comparatively slow process. Under these conditions, there are two main causes that may give raise to systematic errors during measurements of the electrical conductivity. There is at first the transient skin effect after the switching-on, which has a strong influence, but only during the first few τ's [see Eq. (36)]. Then there remains a steady-state field nonuniformity, the origin of which is discussed at the end of Section 3. Its influence is negligible as long as the experimental conditions result in a small ε [see Eq. (37)]. This condition, however, might be violated for very high externally applied voltages during submicrosecond pulse-heating experiments.

Besides the above-mentioned time scale of Eq. (6), which is intimately connected with the electric field equation, Eq. (5), there exists another, larger time scale, which is characteristic for the temperature field equation, Eq. (11), on which heat conduction takes place. The real, steady-state long-time behavior of the electric field occurs on this scale. Since heat conduction flattens the electrical conductivity gradient across the wire, it possibly also reduces the existing field nonuniformity. But its real influence on electrical conductivity measurement is still unknown. Another open problem that can probably be estimated by analytical methods concerns the corresponding influence of the enlargement of the cross section of the wire due to the temperature-dependent volume expansion.

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